Fractal dimension

The fractal dimension is a measure of the complexity of a fractal object, quantifying how the detail in the fractal pattern changes with changes in the scale at which it is measured. Unlike classical geometric objects like lines, squares, or cubes, which have integer dimensions (1, 2, and 3, respectively), fractals often have non-integer dimensions. This non-integer dimension is one of the key characteristics that distinguish fractals from regular geometric shapes.

The concept of fractal dimension was introduced by the mathematician Benoît Mandelbrot to describe the irregular and self-similar structures observed in fractals. The fractal dimension provides a way to measure the degree of self-similarity and complexity of fractal patterns across different scales.

There are several methods for calculating the fractal dimension of a fractal object, depending on its nature and the available data:

Box-Counting Method: This method involves covering the fractal object with boxes of varying sizes and counting the number of boxes needed to cover it at each size. The fractal dimension is then calculated based on how the number of boxes changes as the box size decreases.

Hausdorff Dimension: This dimension is a more general concept that applies to all metric spaces, not just fractals. It measures the “density” of points in a space by considering how the distance between points behaves as the scale changes. For fractals, the Hausdorff dimension often coincides with the fractal dimension obtained through other methods.

Correlation Dimension: This method is based on the correlation integral, which quantifies the probability that points in a space are close to each other as a function of distance. The correlation dimension provides information about the distribution of points in the fractal space.

Information Dimension: This dimension measures the rate of growth of the number of distinct patterns as the scale changes. It is closely related to the concept of entropy and provides insights into the complexity of the fractal pattern.

The fractal dimension is a valuable tool in various fields, including mathematics, physics, biology, and computer science, as it helps characterize and analyze complex structures and phenomena that exhibit self-similarity across different scales.

Fractal dimension in neurosurgery

- Intraoperative superb microvascular ultrasound imaging in glioma: novel quantitative analysis correlates with tumour grade
- Brain structural alterations in vestibular schwannoma beyond tinnitus and hearing loss
- Microvascular heterogeneity exploration in core and invasive zones of orthotopic rat glioblastoma via ultrasound localization microscopy
- Electroencephalographic differences between waking and sleeping periods in patients with prolonged disorders of consciousness at different levels of consciousness
- An Early Progression Biomarker in Glioblastoma: Microcirculatory Heterogeneity on Ultrasound Localization Microscopy
- Diabetes mellitus exacerbates changes in white matter hyperintensity shapes and volume: A longitudinal study

- Computational Fractal-Based Neurosurgery
- Study of prediction model for high-grade meningioma using fractal geometry combined with radiological features

Fractal analysis has emerged as a powerful tool for characterizing irregular and complex patterns found in the nervous system. This characterization is typically applied by estimating the fractal dimension (FD), a scalar index that describes the topological complexity of the irregular components of the nervous system, both at the macroscopic and microscopic levels, that may be viewed as geometric fractals. Moreover, temporal properties of neurophysiological signals can also be interpreted as dynamic fractals. Given its sensitivity for detecting changes in brain morphology, FD has been explored as a clinically relevant marker of brain damage in several neuropsychiatric conditions as well as in normal and pathological cerebral aging. In this sense, evidence is accumulating for decreases in FD in Alzheimer's disease, frontotemporal dementia, Parkinson's disease, multiple sclerosis, and many other neurological disorders. In addition, it is becoming increasingly clear that fractal analysis in the field of clinical neurology opens the possibility of detecting structural alterations in the early stages of the disease, which highlights FD as a potential diagnostic and prognostic tool in clinical practice [1].

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Díaz Beltrán L, Madan CR, Finke C, Krohn S, Di Ieva A, Esteban FJ. Fractal Dimension Analysis in Neurological Disorders: An Overview. Adv Neurobiol. 2024;36:313-328. doi: 10.1007/978-3-031-47606-8_16. PMID: 38468040.